HOLD-UP AND FLOODING IN A VIBRATING PLATE EXTRACTOR

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The mean volumetric hold-up of the dispersed phase and the limiting flow rates of phases in five sizes of experimental vibrating plate extractors VPE were measured and the results compared with the relations of Richardson and Zaki, Míšek and Ishii and Zuber for the relative phase velocity. The experiments were performed with the system toluene–water, toluene dispersed. All the relations were found to describe the dependence of hold-up on flow rates very well. They also allowed a satisfactory estimation of the limiting phase velocities. Within the range of the experimental conditions the characteristic velocity as well as the limiting phase velocities could be approximated by linear functions of the frequency of vibrations. For the columns studied no influence of column size on limiting velocities was detected. The deviations of individual columns from geometrical similarity, however, brought about an appreciable effect both on the hold-up and on the flooding rates. This phenomenon has been explained in terms of longitudinal hold-up profiles, the shape of which depends on the boundary conditions in the particular column. This explanation was corroborated by case studies using a polydisperse mathematical model which show that even in the presence of hold-up profiles the effect of phase velocities on the mean hold-up can be correlated by means of the investigated relations with constant parameters and that the parameters depend on the hold-up profiles form.

The majority of studies dealing with the relation between the hold-up of dispersed phase and the phase velocities, as well as with the phenomenon of flooding in countercurrent extraction columns of various types assume homogeneous distribution of the hold-up of dispersed phase both in axial and radial directions and neglect the polydisperse nature of the dispersion describing it by a suitable mean drop size. Lapidus and Elgin have shown that under these assumptions a unified description of stationary sedimentation, batch fluidisation, cocurrent and countercurrent flow of disperse and continuous phases with and without supporting grid is possible if the concept of the relative (slip) velocity of the particles with respect to the continuous phase is used

$$u_s = U_d/X + U_c/(1 - X) . \qquad (1)$$

The slip velocity is a result of action of external and internal forces on the continuous and dispersed phases in the gravity field. Its dependence on the particle size, on the density and viscosity of phases, on the hold-up of the dispersed phase and on the particle-to-column diameter ratio is usually expressed as the product of the so called "characteristic velocity" defined as the slip velocity at zero hold-up and of a hold-up correction factor

$$u_s = U_0 \, . \, \Phi(X, \varrho_d, \varrho_c, \mu_d, \mu_c, d/D) \, ; \quad \lim_{X \to 0} \Phi = 1 \, . \qquad (2)$$

For small d/D values U_0 is usually assumed to equal the terminal settling velocity of the particle in the infinite quiescent continuous phase. A general formula for the correction factor ϕ has been proposed by Ishii and Zuber[2]. It holds for a solid, liquid or gaseous dispersed phase and is based on the concepts of mixture density and viscosity ϱ_s, μ_s.

Already in the early studies of the extraction columns hydrodynamics the relations (*1*) and (*2*) for the motion of phases in empty columns were applied to packed, mixed and pulsed columns as well. In such cases one must, of course, expect that both the characteristic velocity U_0 and the correction factor ϕ will be affected by the complex pattern of the continuous phase flow and that the mean drop size will be the result of extensive breakage and coalescence. For the correction factor different forms were proposed. Thus Thornton[3] for the pulsed plate column, Logsdail and coworkers[4] and Strand and coworkers[5] for the rotating disc contactor used the formula

$$\Phi = 1 - X\,. \tag{3}$$

Míšek[6–8] proposed for RDC and ARD extractors and for pulsed plate columns with sieve plates with big hole diameter the formula

$$\Phi = (1 - X)\exp\left[(z - 4{\cdot}1)\,.\,X\right] \tag{4}$$

which originates from Steinour's[9] relation for hindred settling of rigid spheres in laminar regime and in which the correction parameter z expressing the influence of drop coalescence was incorporated. These authors obtained a very good fit of their formulae with their own experimental data, but a comparison of these formulae on a common set of data has not yet been undertaken.

According to Lapidus and Elgin[1] flooding under countercurrent flow can be defined by the following relations

$$(\mathrm{d}U_c/\mathrm{d}X)_F,\ U_d = (\mathrm{d}U_d/\mathrm{d}X)_F,\ U_c = 0\,. \tag{5}$$

Thornton and Míšek applied these relations to the formulae (*3*) and (*4*) and they obtained for the hold-up at flooding

$$\frac{U_{dF}}{U_{cF}} = \frac{2X_F^2}{(1 - 3X_F + 2X_F^2)} \tag{6}$$

and

$$\frac{U_{dF}}{U_{cF}} = \frac{2X_F^2[1 - X_F + (z - 4{\cdot}1)(X_F - X_F^2/2 - 1/2)]}{(1 - X_F)^2\,[1 - 2X_F + (z - 4{\cdot}1)(X_F - X_F^2)]}\,. \tag{7}$$

Recently a number of papers have been published describing expressed longitudinal hold-up profiles in various types of mechanically stirred extraction columns[5,10–12]. The profiles arise not only under the influence of changes of physical properties caused by changes in composition, but also as a result of entrainment of fine droplets, of back pumping of dispersed phase and of breakage and coalescence of drops. It is not very clear to what extent the existence of these profiles may invalidate the lumped parameter models (*3*) to (*7*).

The purport of the present work was to compare some lumped parameter models for hold-up and flooding on a set of data obtained in five vibrating plate columns of the VPE type differing in size and to examine the variability of their parameters with respect to the frequency of plate motion and to the column size.

Applying the formulae of type (2) to the calculation of the slip velocity in mechanically agitated columns with complex geometry one may expect that both the characteristic velocity U_0 and the correction factor Φ will be functions of the intensity of agitation and of relevant geometrical variables. Accordingly it is no longer possible to interpret U_0 as the terminal velocity of drops of mean size. Three formulae were compared each containing two empirical parameters, one of which was U_0. The criterion of the comparison was to what extent these parameters preserved constant values independent of the phase velocities.

The first relation was proposed by Richardson and Zaki[13]

$$u_s = U_0(1 - X)^a . \tag{8}$$

The relation used by Thornton and others is a special case of Eq. (8) ($a = 1$). Míšek's[6] formula was used in the form

$$u_s = U_0(1 - X) \exp(bX) . \tag{9}$$

The third relation used was that of Ishii and Zuber[2] in the special form for liquid–-liquid dispersions

$$u_s = U_0 \frac{(1 - X)^{\varphi_1(\mu_c, \mu_d)}}{1 + c\left[(1 - X)^{\varphi_2(\mu_c, \mu_d)} - 1\right]} , \tag{10}$$

$$\varphi_1(\mu_c, \mu_d) = 1 + 2{\cdot}5 \frac{\mu_d + 0{\cdot}4\mu_c}{\mu_d + \mu_c} ,$$

$$\varphi_2(\mu_c, \mu_d) = \frac{6}{7}\left(\frac{1}{2} + 2{\cdot}5 \frac{\mu_d + 0{\cdot}4\mu_c}{\mu_d + \mu_c}\right) .$$

The empirical parameter c in Eq. (10) corresponds to the following function of the parameter ψ originally used in the Ishii and Zuber's model for infinite medium

$$c = \frac{\psi(r^*)}{1 + \psi(r^*)} ; \quad \psi(r^*) = 0{\cdot}55\left[(1 + 0{\cdot}08r^{*3})^{4/7} - 1\right]^{3/4} ;$$

$$r^* = \frac{d}{2}\left[\varrho_c\, g |\Delta\varrho / \mu_c^2\right]^{1/3} .$$

The relations (8) to (10) may be written in linear form and their parameters U_0,

a, b, c can be determined using the least squares method

$$\ln u_s = a \,.\, \ln (1 - X) + \ln U_0 \tag{11}$$

$$\ln (u_s/(1 - X)) = bX + \ln U_0 \tag{12}$$

$$\frac{1}{(1 - X^{\varphi_2}) - 1} = \frac{U_0}{u_s} \frac{(1 - X)^{\varphi_1}}{(1 - X)^{\varphi_2} - 1} - c \,. \tag{13}$$

EXPERIMENTAL

The measurement of hold-up and flooding was done in five vibrating plate columns of the VPE type. The dimensions of these columns are given in Table I. The columns with rectangular cross section were used for capacity reasons. On both ends the columns were provided with enlarged settling sections. The length of columns 1 and 2 was 4 m, the other columns were 2 m long. The length of the set of plates, however, was 2 m in all columns.

The arrangement of the columns with circular and rectangular cross sections is shown in Fig. 1. In both cases perforated plates with downcomers were used. Such arrangement of openings causes the dispersed phase to flow preferentially through the perforations and the continuous phase to flow through the downcomers. In cylindrical columns the downcomers were cylinders directed against the dispersed phase flow, in rectangular columns the downcomers had the form of a rectangular slit between the column wall and a vertical partition lining the plate edge. As in the neighbouring plates the downcomers were situated opposite to the column axis, a cross flow of continuous and dispersed phases between the plates occurred.

The organic phase being dispersed, the interface was situated in the upper settler. In the columns of 4 m length with 2 m long sets of plates the upper plate was in the midst of the column and the interface was kept 10 cm above it.

TABLE I

Main dimensions of models of vibrating plate extractor VPE. Common dimensions: length of set of plates: 200 cm; plate spacing: 10 cm; diameter of perforations: 0·3 cm; free area of perforations: 10%; free area of downcomers: 15%; hight of downcomers: 4 cm

Model	Inner diam. or dimensions cm	Column length cm	Diameter of settlers cm	Hight of settlers cm
1	4·79	429	12	27
2	8·50	408	18	40
3	8·50	200	18	40
4	5 × 15	200	24	40
5	7·5 × 22·5	200	24	40

Accordingly the geometry of individual columns was diverse in some respects. However, a number of parameters which were thought essential for preserving the geometrical similarity of the sets of plates, were kept constant. These were: the plate spacing, the diameter of perforations, their specific free area, the specific free area of the downcomers and their height. Thus, under the assumptions of radially homogeneous flow through the perforations, of little effect of variable length of the cross flow path for continuous phase within the given limits, as well as of negligible influence of the variable end effects at the given number of plates in the set, one could anticipate good modelling of the specific volumetric hold-up and of the limiting velocities in the columns in question.

All measurements were carried out with the system toluene–water, toluene dispersed. The working temperature was 25 ± 0·5°C. For the hold-up measurements the method of sudden interruption of the entering and leaving streams was applied; the overall accuracy of the method was in most cases ±5% rel. Independent variables were the phase velocities and the vibrational frequency. In experiments in columns 1 and 2 the flooding point was also measured.

RESULTS AND DISCUSSION

Fig. 2 shows an example of typical hold-up measurements in dependence on the sum of phase velocities for four frequency values. From such results the parameter values of relations (*8*) to (*10*) were obtained by the least squares method. They are

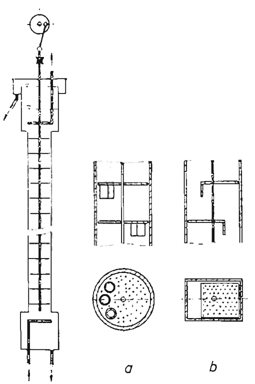

FIG. 1

Diagrammatic drawing of vibrating plate column: *a* Circular cross section, *b* rectangular cross section

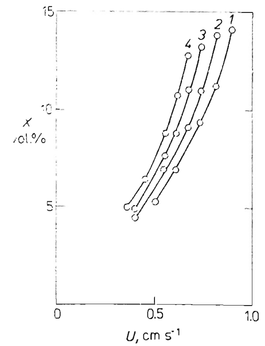

FIG. 2

Dependence of specific volumetric hold-up on sum of phase velocities, column 3: *1* $f = 2$ Hz; *2* $f = 2·5$ Hz; *3* $f = 3$ Hz; *4* $f = 3·5$ HZ summarized in Table II in which also the relative mean square deviation of the slip velocity for the individual formulae and conditions is presented.

From Table II follows in the first place, that all the three relations examined correlate the experimental data with high accuracy; only in two cases the mean error exceeds 2%. Thus the error of correlation may be considered comparable with that of the measurements. This statement is also confirmed by the fact that for individual measurements the error values for all three relations usually move in a common way. The error of Eq. (*9*) is the lowest one, but the other two formulae are only a little less accurate. According to the Fischer test, in which the variances of relations (*8*) and (*10*) were compared with that of Eq. (*9*), the relation (*8*) is not worse on the 10% significance level, but the relation (*10*) is worse even on the 1% level. On the whole one can say, however, that in the range of conditions studied the three formulae can be taken for equivalent, as long as their parameters are considered purely empirical, adjustable quantities.

It is remarkable that the values of U_0 for the different formulae differ very little and randomly from each other. Consequently the relations investigated have similar asymptotic behaviour. The drop size was not measured, but when assessed on the assumption of the characteristic velocity being equal to the terminal velocity of single drop the mean drop size assumed the values in the range 0·4 to 0·9 mm, which seems plausible.

The values of the parameter a in the formula (*8*) are within the range 1·3 to 3·5. Thus the value 3·0 corresponding to the relations for a in the work of Richardson and Zaki[13] lies near to the upper limit of this range. The value 1 used in some works[3-5] is definitely too low. The absolute values of the exponent b in the formula (*9*) are markedly lower than 4·1 which means, according to Eq. (*4*), that the value of the "coalescence factor" z significantly differs from zero. According to Ishii and Zuber[2] for drops of 1 mm diameter the value of ψ should be 1·36. This value lies within the range of the parameter values in Table II, but among these values are also negative ones, which are incompatible with the theoretical relation $\psi(r^*)$.

Let us draw attention to the effect of frequency on the parameter values. According to Fig. 3 in the given frequency range the characteristic velocity can be approximated by a linear function of frequency. As expected it decreases with increasing value of the argument. Its mean values for columns of 4 m length differ significantly from those found in columns of 2 m length. The influence of the frequency on the second parameters of relations (*8*) to (*10*) is less conspicuous and often nonmonotonous. The most sensitive parameter to this variable is ψ. It can be observed, however, that the largest deviation from the mean value appears simultaneously for all parameters at a given frequency. Accordingly it is a property of the data and not of the relations.

Similarly to the mean values of U_0, the means with respect to frequency of the second parameters differ for the individual columns, and the differences are even bigger than those of the characteristic velocity. These differences are to be attributed to the variation in column geometry, mainly to the diverse conditions at the column ends. The lesser sensitivity of U_0 to These factors indicates, that the end effects influence

TABLE II

Parameters of hold-up correlations $A = 0{\cdot}2$ cm; $U_d/U_c = 1{\cdot}15$

Model	f	Eq. (*8*)		Eq. (*9*)		Eq. (*10*)		$\sigma(u_s) \times 10^2$		
	Hz	U_0 cm s^{-1}	a	U_0 cm s^{-1}	b	U_0 cm s^{-1}	ψ	Eq. (*8*)	Eq. (*9*)	Eq. (*10*)
1	2·5	5·17	1·98	5·24	−1·18	5·07	0·79	1·85	1·80	2·04
	3·0	3·96	1·43	3·99	−0·53	3·97	1·96	0·92	0·77	1·56
	3·5	3·27	1·44	3·31	−0·57	3·27	2·05	1·09	0·92	1·90
	4·0	2·89	1·34	2·91	−0·43	2·95	1·96	1·93	1·86	2·61
	mean value	*3·82*	*1·55*	*3·86*	*−0·68*	*3·82*	*1·69*	*1·45*	*1·34*	*2·03*
2	2·0	4·73	2·21	4·79	−1·41	4·78	0·20	0·92	0·75	1·19
	2·5	4·20	2·08	4·24	−1·25	4·22	0·37	0·67	0·59	0·86
	3·0	3·75	1·88	3·78	−1·01	3·71	0·83	0·45	0·50	0·46
	3·5	2·71	1·23	2·72	−0·28	2·73	3·22	0·95	0·92	1·22
	mean value	*3·85*	*1·85*	*3·88*	*−0·99*	*3·86*	*1·15*	*0·75*	*0·69*	*0·93*
3	2·0	6·32	3·28	6·39	−2·52	6·39	−0·35	0·20	0·10	0·06
	2·5	5·73	3·17	5·78	−2·39	5·75	−0·30	0·46	0·51	0·49
	3·0	5·43	3·43	5·49	−2·67	5·46	−0·38	0·86	0·87	6·88
	3·5	4·78	3·08	4·82	−2·28	4·75	−0·23	0·93	0·96	1·01
	mean value	*5·57*	*3·24*	*5·62*	*−2·47*	*5·59*	*−0·32*	*0·61*	*0·61*	*0·61*
4	2·0	6·59	2·11	6·63	−1·25	6·60	0·34	1·31	1·25	1·36
	2·5	6·28	2·02	6·32	−1·16	6·36	0·32	1·63	1·61	1·91
	3·0	2·21	2·21	6·06	−1·30	5·97	0·43	1·32	1·32	1·38
	3·5	5·51	1·88	5·53	−0·96	5·47	0·84	0·69	0·72	0·71
	mean value	*6·11*	*2·06*	*6·14*	*−1·17*	*6·10*	*0·48*	*1·24*	*1·23*	*1·34*
5	2·0	6·96	1·80	6·97	−0·86	6·81	1·60	1·19	1·20	1·29
	2·5	5·83	1·75	5·84	−0·80	5·81	1·00	0·34	0·34	0·34
	3·0	5·16	1·63	5·17	−0·68	5·10	1·69	0·68	0·70	0·79
	3·5	4·10	1·26	4·40	−0·28	4·38	3·50	0·42	0·43	0·42
	mean value	*5·59*	*1·61*	*5·59*	*−0·66*	*5·53*	*1·95*	*0·66*	*0·67*	*0·71*
						$\bar{\sigma}(u_s) \times 10^2$		0·94	0·91	1·12

primarily the shape of the hold-up profile, which becomes important at higher mean hold-ups having a somewhat lesser effect on the asymptotic parameter U_0. Accordingly one can assume that the existence of the hold-up profiles affects primarily the value of the second parameter of the formula in question. The influence of the end effects, mainly of the length of the column, on the characteristic velocity may be explained by its nature as an extrapolated quantity.

When comparing the mean paramter values for the individual columns, no systematic effect of the characteristic radial dimension can be detected. Accordingly it may be expected that scaling up of cylindrical columns in the given range of diameter values will proceed with constant specific throughout.

During the hold-up measurements in columns 1 and 2 limiting phase velocities were measured. This quantity can also be predicted using relations (*8*) to (*10*) and conditions (*5*). The agreement of the measured and predicted values may then be interpreted as the confirmation of the validity of the formula in question up to the flooding point. Such prediction was made by means of Eq. (*9*) or (*7*). Using the hold-ups at flooding X_F, calculated by Eq. (*7*) the limiting velocities were obtained from the equations

$$U_{cF} = U_0(1 - X_F)^2[1 - (2 - b)X_F - bX_F^2]\exp(bX_F), \qquad (14)$$
$$U_{dF} = U_0x_F^2(1 - X_F)[2 - b(1 - X_F)]\exp(bX_F).$$

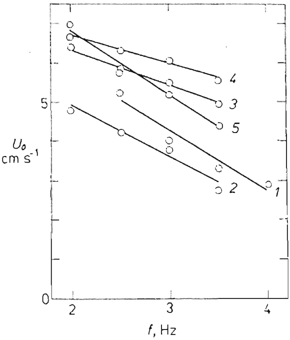

FIG. 3

U_0 *vs* f for columns 1 to 5

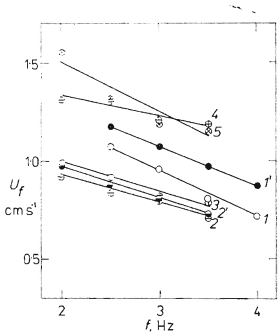

FIG. 4

U_F *vs* f for columns 1 to 5. Numbers with prime denote measured values, the rest calculated by Eqs (*7*) and (*8*)

The results are shown in Fig. 4, where the experimental values for columns 1 and 2 are also depicted. The drop of the limiting velocity with frequency is again approximately linear, as it was in the case of the characteristic velocity. For column 2 the experimental values coincide with the calculated ones, for column 1 they are 10 to 20% higher.

Case Studies by Polydisperse Model

In the present work an assumption was made that the deviations in the parameter values of the individual columns are caused by the variation of the hold-up profile shapes initiated in turn by the differences in the geometry of the column ends. As formulae with constant parameters were successfully applied to these hold-up *vs* phase velocity data, one must, however, also assume the formulae to be flexible enough to simulate this data with constant values of adjustable parameters even in the presence of axial hold-up profiles.

These assumptions could not be verified experimentally, because the hold-up profiles were not measured in the present work. To corroborate them case studies by the polydisperse hydrodynamic model of a vibrating plate extractor proposed by Sovová[14] were undertaken. This distributed parameter model incorporates the effects of breakage and coalescence of drops, of the back and forward mixing of the dispersion and of the entrainment of fine droplets by the continuous phase. In its present form it is fitted to the Karr column and the values of its parameters were obtained by

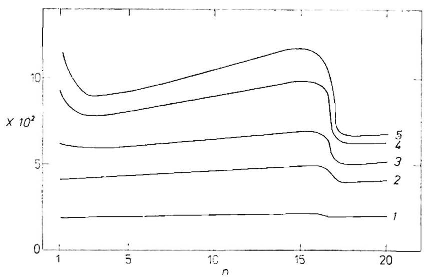

FIG. 5

Simulated hold-up profiles in Karr column with 1,2-dichloroethane–water, organic phase dispersed. $U_d/U_c = 0{\cdot}5$; $A = 0{\cdot}5$ cm; $f = 3{\cdot}5$ Hz; *1* $U = 0{\cdot}006$ cm s^{-1}; *2* $U = 0{\cdot}012$ cm s^{-1}; *3* $U = 0{\cdot}015$ cm s^{-1}; *4* $U = = 0{\cdot}018$ cm s^{-1}; *5* $U = 0{\cdot}0195$ cm s^{-1}

evaluating the data of Jiřičný and Procházka[12] measured in this column with the system 1,2-dichloroethane–water, the organic phase dispersed. The model also accounts for the inhomogeneity of the hold-up distribution within individual column stages, *i.e.* that with the exception of very intense mixing regimes the hold-up within the stage increases in the direction of the dispersed phase flow.

The hold-up profiles for a column with 20 stages and with the continuous phase fed into the 17th stage were calculated. Five values of the phase velocity and four frequency levels were chosen. The changes of the hold-up profile shape with gradual increase of the phase velocity are depicted in Fig. 5. The mean hold-up values for such profiles are plotted in the coordinates corresponding to Eq. (*12*) in Fig. 6. Apparently these results comply with Eq. (*9*).

The second series of computations concerned an infinite column under conditions of no drop coalescence in which the free area of plates was gradually decreased. Under these conditions an overall hold-up profile cannot build up, as the breakage of drops must eventually stop and the condition of an infinite column precludes the influence of its ends on the drop transport. Nevertheless the hold-up profiles within the stages may still exist. They will become steeper with the free area of plates decreasing because of the increasing velocity of the continuous phase in the plate

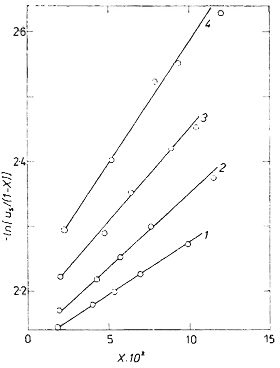

FIG. 6

ln $u_s/(1 - X)$ *vs* X according to Eq. (*4*). $A = 0.5$ cm: *1* $f = 2.5$ Hz; *2* $f = 3$ Hz; *3* $f = 3.5$ Hz; *4* $f = 4$ Hz

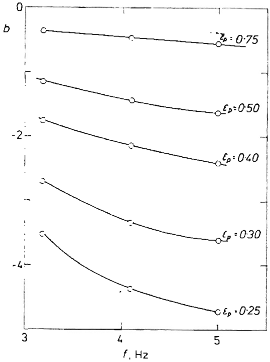

FIG. 7

Dependence of b on free area of plate and frequency for infinite column and no coalescence. $A = 0.5$ cm perforations. The stepwise changes of the hold-up across the plates in the presence of the back pumping mechanism will then cause an increased dispersed phase transport in the direction of its main flow. Fig. 7 shows that this process has a similar effect on the value of b as an increasing coalescence rate would exert. This knowledge casts a new light on the complex physical nature of the so called coalescence factor z in Eq. (*4*). In spite of the fact that in the present case only the effect of the hold-up profile within the stages was examined, it is apparent that an overall profile would exert a similar influence.

LIST OF SYMBOLS

a	parameter in relation (*8*)
A	amplitude of plate vibrations, cm
b	parameter in relation (*9*)
$c = \psi/(1 + \psi)$	parameter in relation (*10*)
d	mean drop diameter, cm
D	inner column diameter, cm
f	frequency of plate vibrations, Hz
g	acceleration of gravity, cm s^{-2}
r^*	dimensionless drop radius
u_s	relative velocity of drops with respect to the continuous phase (slip velocity), cm s^{-1}
U_i	superficial velocity of phase i, cm s^{-1}
U_0	characteristic velocity, cm s^{-1}
X	specific volumetric hold-up of dispersed phase
z	coalescence parameter
ε_p	specific free area of plate
μ	dynamic viscosity, g cm^{-1} s^{-1}
ϱ	density, g cm^{-3}
$\Delta\varrho$	absolute value of difference of phase densities, g cm^{-3}
σ	mean square deviation
ϕ	correction factor on finite hold-up

Subscripts

c	continuous phase
d	dispersed phase
F	flooding

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